Adaptive viscosity methods for the computation of turbulent flows

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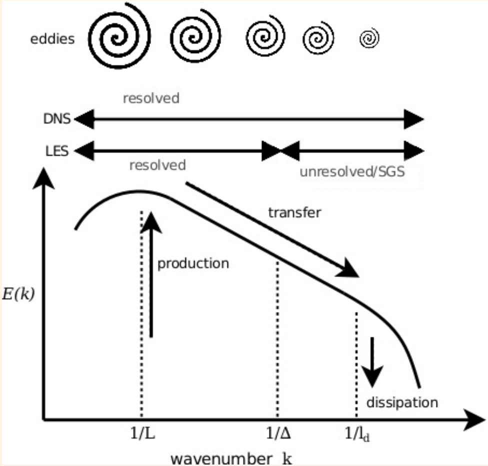
Overview

Introduction

Turbulent energy cascade

- Turbulent motions range from a macroscale (energy supplied), to a microscale where energy is gradually dissipated.
- The largest vortices are created by instabilities of the mean flow and they are also subject to different types of instabilities and may break into smaller ones
- The smaller vortices abide by the same rules and may break again
- Energy is eventually dissipated by viscosity in the smallest scales, where the local Reynolds number $\mathrm{Re} \approx 1 \implies$ Kolmogorov scale



Energy transfer in compressible flows

- In compressible flows the turbulent fluctuations can be split into a compressible and an incompressible part
- The incompressible part follows the turbulent energy cascade, but compressible turbulence is much more complex since acoustic and entropy fluctuations are also taking place, and these fluctuations are neglected by incompressible turbulence models.
- Using a ”no model” approach with a numerical method with the ability to solve an increased range of scales, would make possible to account for these fluctuations
- The simulation of turbulent flows **without an explicit model** is of great interest since no additional hypotheses are introduced

Navier–Stokes equations for compressible flows

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} + \frac{\partial \mathbf{H}}{\partial z} = \frac{\partial \mathbf{F_v}}{\partial x} + \frac{\partial \mathbf{G_v}}{\partial y} + \frac{\partial \mathbf{H_v}}{\partial z}$$

$$\mathbf{U} = \big(\rho, \rho u, \rho v, \rho w, \rho E\big)^T$$

$$\mathbf{F} = \big(\rho u, \rho u^2 + p, \rho u v, \rho u w, u(\rho E + p)\big)^T \qquad \mathbf{F_v} = \big(0, \tau_{xx}, \tau_{xy}, \tau_{xz}, \tau_{xk} u_k - q_x\big)^T$$

$$\mathbf{G} = \big(\rho v, \rho u v, \rho v^2 + p, \rho v w, v(\rho E + p)\big)^T \qquad \mathbf{G_v} = \big(0, \tau_{xy}, \tau_{yy}, \tau_{yz}, \tau_{yk} u_k - q_y\big)^T$$

$$\mathbf{H} = \big(\rho w, \rho u w, \rho v w, \rho w^2 + p, w(\rho E + p)\big)^T \qquad \mathbf{H_v} = \big(0, \tau_{xz}, \tau_{yz}, \tau_{zz}, \tau_{zk} u_k - q_z\big)^T$$

$$\mathbf{V} = (u, v, w)^T \qquad \mathbf{q} = -\kappa \,\mathrm{grad}\, T \qquad \kappa = \frac{\gamma R \mu}{\Pr(\gamma - 1)}$$

$$\boldsymbol{\tau} = -\tfrac{2}{3}\mu(\mathrm{div}\,\mathbf{V})\mathbf{I} + \mu\left(\mathrm{grad}\,\mathbf{V} + (\mathrm{grad}\,\mathbf{V})^T\right)$$

Introduction

- Two fundamental questions:
  - If we accept that the Navier-Stokes equations model the dynamic behaviour of a flow, should we change the equations depending on the laminar or turbulent regime?
  - Can turbulence modeling be considered independently of the numerical method?

Introduction

- The numerical method must be designed to mimic the physics, to solve all of the energy transfer modes.
- We propose an implicit SGS model based on adaption of the intrinsic viscosity of the numerical scheme.

- GENERAL APPLICABILITY
  - The proposed method can simulate laminar and transitional flows
  - It is applicable to complex flows (multiphase, non-Newtonian,...) where no turbulence models even exist.

Introduction

- BASIC IDEA
  - The basic idea is to locally detect if the flow is under-resolved or not. If it is, the numerical method increases its dissipation. If it is not, the numerical method decreases its dissipation.
  - This is an automatic procedure to obtain the best possible solution for the chosen numerical method in a given grid.
  - It is a numerics-based approach, but physically calibrated

Adaptive-dissipation schemes for the numerical simulation of compressible turbulent flows

Automatic Dissipation Adjustment method

- Numerical errors + insufficient spatial resolution $\Longrightarrow$ under-resolved flow
- Under-resolved flow $\Longleftrightarrow$ not all the flow scales are captured $\Longrightarrow$ significant high-frequency content
- Must remove high-freq content to mimic the energy cascade. Implicit LES

Automatic Dissipation Adjustment method

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- Must remove high-freq content to mimic the energy cascade. Implicit LES
- Regulates the local dissipation adaptively based on some measurement of how well-resolved is the flow
- This measurement is called the local energy ratio (ER) [1]. It is a ratio of the high-frequency velocity components using two different filters with different filtering widths.

$$\mathrm{ER} = \frac{\sum \left(u_i - \widetilde{u}_i\right)^2}{\sum \left(u_i - \widehat{u}_i\right)^2}$$

[1] J. A. Domaradzki *et al.* in Theoret. Comput. Fluid Dynamics 15 421–450 (2002)

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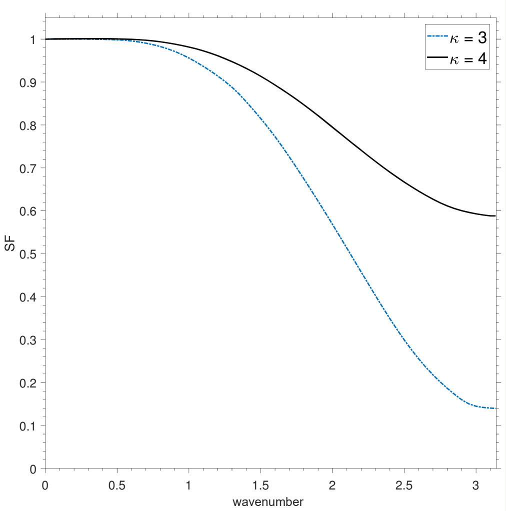
$$\mathrm{ER} = \frac{\sum \left(\rho u_i - \widetilde{\rho u}_i\right)^2}{\sum \left(\rho u_i - \widehat{\rho u}_i\right)^2}$$

- To obtain $\widetilde{\rho u}_i$ and $\widehat{\rho u}_i$, MLS-based (or kernel/based) filters were used with different parameters κ equal to 4 and 3, respectively

[1] J. A. Domaradzki *et al.* in Theoret. Comput. Fluid Dynamics 15 421–450 (2002)

Automatic Dissipation Adjustment method



WENO5 ADA scheme

- Decreasing the dissipation of the scheme can make it *unstable* when in the presence of shocks
- In order to ensure stability, the *a posteriori* procedure is employed.

MOOD framework

- An *a posteriori* detection criterion based on the Multidimensional Optimal Order Detection (MOOD) method [1].
- A candidate solution $\mathbf{U}^*$ is computed with the less dissipative scheme using the solution and the ADA values from the previous time step $\mathbf{U}^{\mathrm{RK}}$.
- A series of tests are performed on the candidate solution (detectors) to check whether or not the candidate solution is valid.
- If it is not, the solution is recomputed locally with a more stable scheme (WENO, 1st-order reconstruction of Riemann states,..).
- The advantage of the *a posteriori* detection is that it does not rely on "guesses" to predict the formation of shocks, the shock once it appears is properly identified and treated accordingly.

[1] S. Clain, S. Diot and R. Loubère, JCP 230(10),(2011)

MOOD framework

- The following detectors are employed:
  - Physical Admissibility Detection (PAD)[1]: all points must have positive density and pressure at all times. It also has to detect `NaN` values.

$$\rho(i) > 0 \text{ and } p(i) > 0 \ \forall i$$

  - Numerical Admissibility Detection (NAD)[2]: relaxed version of the Discrete Maximum Principle (DMP).

$$\min_{\mathbf{y}\in\mathcal{V}_i} \left(\mathbf{U}^n\left(\mathbf{y}\right)\right) - \delta \leqslant \mathbf{U}^*(\mathbf{x}) \leqslant \max_{\mathbf{y}\in\mathcal{V}_i} \left(\mathbf{U}^n\left(\mathbf{y}\right)\right) + \delta$$

$$\delta = \max\left(10^{-4}, 10^{-3} \cdot \left(\max_{\mathbf{y}\in\mathcal{V}_i} \left(\mathbf{U}^n\left(\mathbf{y}\right)\right) - \min_{\mathbf{y}\in\mathcal{V}_i} \left(\mathbf{U}^n\left(\mathbf{y}\right)\right)\right)\right)$$

[1] J. Figueiredo and S. Clain, SYMCOMP 2015

[2] M. Dumbser, O. Zanotti, R. Loubère and S. Diot in JCP, 278, (2014)

WENO5 ADA scheme

- The ADA mechanism is implemented into the WENO5 scheme via a multiplicative coefficient $\varepsilon \in [0,1]$ that regulates the amount of dissipation introduced by the scheme at each Runge-Kutta stage.

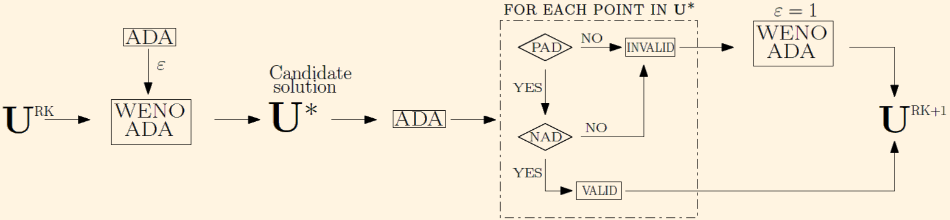
$$\varepsilon_{k+1} = \begin{cases} \downarrow \varepsilon_k & \text{ER} \leq \varphi^L \\ \uparrow \varepsilon_k & \text{ER} \geq \varphi^H \\ = \varepsilon_k & \text{case else} \end{cases} \quad \Bigg| \quad \phi = 0.05$$

$$\mathbf{F}_{i+1/2} = \begin{pmatrix} \text{central} \\ \text{scheme} \end{pmatrix} + \varepsilon_{i+1/2} \times \begin{pmatrix} \text{dissipative} \\ \text{part} \end{pmatrix}$$

$$\varepsilon_{i+1/2} = \max\left[\varepsilon_i \ , \ \varepsilon_{i+1}\right]$$

- If PAD or NAD flag the solution as invalid, recalculate with the full WENO5 scheme, corresponding to $\varepsilon = 1$

Overall WENO-ADA framework



δ-SPH-ADA scheme

- We apply the ADA method for weakly compressible SPH formulations.
- In particular we chose the δ^+-LES-SPH method.

$$\begin{cases} \dfrac{D\rho_i}{Dt} = -\rho_i \sum_j (\boldsymbol{u}_j - \boldsymbol{u}_i) \cdot \nabla_i W_{ij} V_j + h c_0 \sum_j \delta_{ij} \boldsymbol{\mathcal{D}}_{ij} \cdot \nabla_i W_{ij} V_j \\ \dfrac{D\boldsymbol{u}_i}{Dt} = \boldsymbol{g}_i - \dfrac{1}{\rho_i} \sum_j (p_i + p_j) \nabla_i W_{ij} V_j + h c_0 \dfrac{\rho_0}{\rho_i} \sum_j \alpha_{ij} \pi_{ij} \nabla_i W_{ij} V_j \\ \dfrac{D\boldsymbol{r}_i}{Dt} = \boldsymbol{u}_i; \; p_i = c_0^2 (\rho_i - \rho_0); \; \boldsymbol{r}_i^* = \boldsymbol{r}_i + \delta \boldsymbol{r}_i \end{cases}$$

With $\alpha = \alpha_{Physical} + \alpha_{Turbulent}$.

- $\boldsymbol{r}^*$ is the modified position after the particle shifting, and $\delta \boldsymbol{r}$ is applied for tensile instability control [1].

[1]: P.N. Sun, A. Colagrossi, S. Marrone, M. Antuono, A.M. Zhang, Multi-resolution Delta-plus-SPH with tensile instability control: Towards high Reynolds number flows,Computer Physics Communications 224, 63-80 (2018).

δ-SPH-ADA scheme

- Following the same idea used before, we introduce the ϵ parameter in the dissipative part of the momentum equations

$$\begin{cases} \dfrac{D\rho_i}{Dt} = -\rho_i \sum_j (\boldsymbol{u}_j - \boldsymbol{u}_i) \cdot \nabla_i W_{ij} V_j + hc_0 \sum_j \delta_{ij} \boldsymbol{\mathcal{D}}_{ij} \cdot \nabla_i W_{ij} V_j \\ \dfrac{D\boldsymbol{u}_i}{Dt} = \boldsymbol{g}_i - \dfrac{1}{\rho_i} \sum_j (p_i + p_j) \nabla_i W_{ij} V_j + hc_0 \dfrac{\rho_0}{\rho_i} \sum_j \alpha_{ij} \pi_{ij} \nabla_i W_{ij} V_j \\ \dfrac{D\boldsymbol{r}_i}{Dt} = \boldsymbol{u}_i; \; p_i = c_0^2 (\rho_i - \rho_0); \; \boldsymbol{r}_i^* = \boldsymbol{r}_i + \delta \boldsymbol{r}_i \end{cases}$$

- With $\alpha = \alpha_{Physical} + \alpha_{Turbulent} = \alpha_{Physical} + \epsilon_{ij}$.
- With $\epsilon_{ij} = \frac{\epsilon_i + \epsilon_j}{2}$.
- Kernel is used as filter instead MLS.
- The rest of the configuration is the same than for the WENO case (without MOOD).

δ-SPH-ADA scheme

- In order to automatically adjust the parameter, we define the value ϵ_i associated to a particle i as

$$\begin{cases} ER_i < 0.5, & \epsilon_i = \max[(\epsilon_i - \phi), 0] \\ ER_i > 0.55, & \epsilon_i = \min[(\epsilon_i + \phi), 0.01] \\ 0.5 \leq ER_i \leq 0.55, & \epsilon_i \text{ does not change} \end{cases}$$

  - A value of $\phi = 0.001$ is used, to adjust the value of ϵ_i continuously and gradually. The maximum value of ϵ_i is determined following [1].

[1]: M. Antuono, A. Colagrossi, S. Marrone. Numerical diffusive terms in weakly-compressible SPH schemes. 399 Computer Physics Communications, 183, 2570–2580 (2012).

Numerical examples

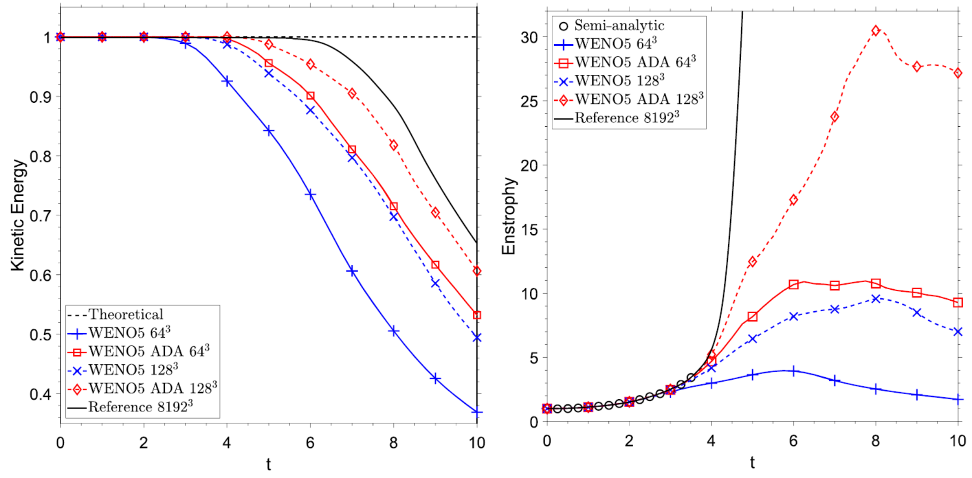
Incompressible isotropic Taylor-Green vortex ($\mathrm{Re} \to \infty$)

- 3D Euler equations ($\mathrm{Re} \to \infty$). Initial conditions

$$\rho(x, y, z, 0) = 1$$
$$u(x, y, z, 0) = \sin(x)\cos(y)\cos(z)$$
$$v(x, y, z, 0) = -\cos(x)\sin(y)\cos(z)$$
$$w(x, y, z, 0) = 0$$
$$p(x, y, z, 0) = 100 + \frac{1}{16}[(\cos(2x) + \cos(2y))(2 + \cos(2z)) - 2]$$

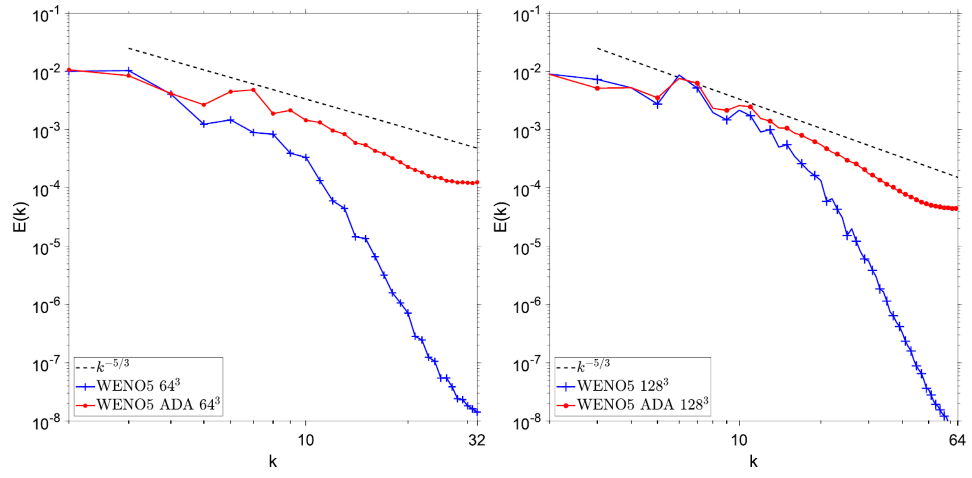
- Domain: $[0,\ 2\pi]^3$
- Final time of simulation: $t = 10$
- Number of points in mesh: $64^3, 128^3$

Incompressible isotropic Taylor-Green vortex Re_∞



Kinetic energy and Enstrophy time evolution

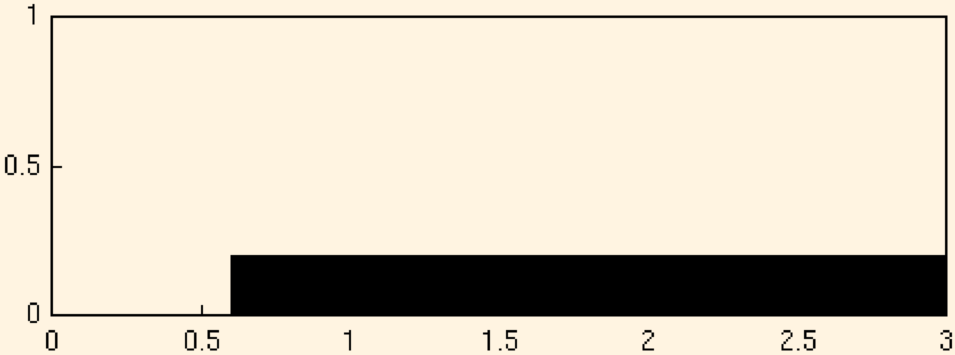
Incompressible isotropic Taylor-Green vortex Re_∞



Energy spectra at $t = 10$

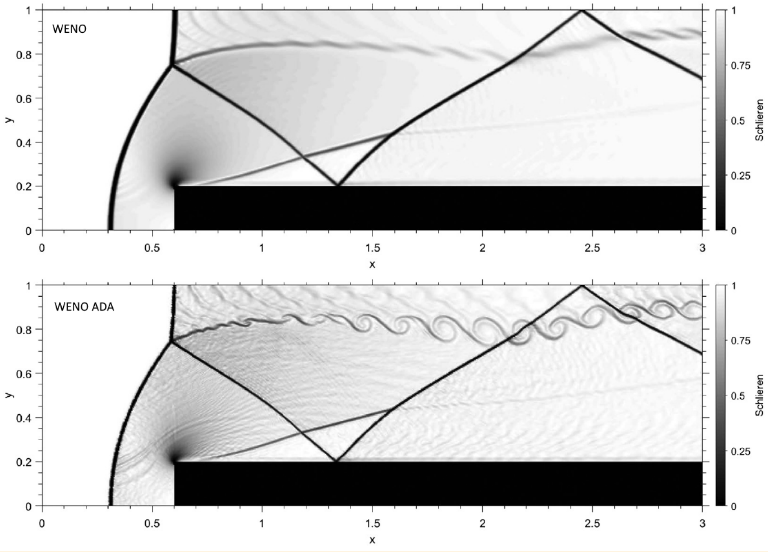
Mach 3 tunnel with a step

- 2D Euler equations. Initial conditions



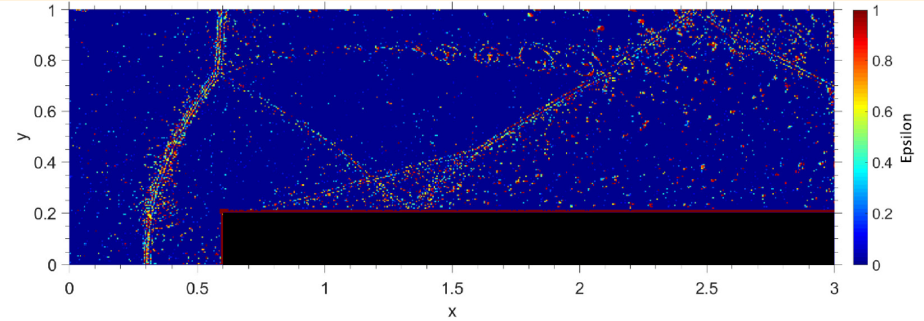
- Domain: [0, 3]×[0, 1]   Step: [0.6, 3]×[0, 0.2]
- Final time of simulation: $t = 4.0$
- Number of points in mesh: 600×200

Mach 3 tunnel with a step



WENO vs. WENO ADA for a 600×200 mesh.

Mach 3 tunnel with a step



Epsilon values for a 600×200 mesh.

Decay of compressible homogeneous isotropic turbulence. HIT

- Initial conditions

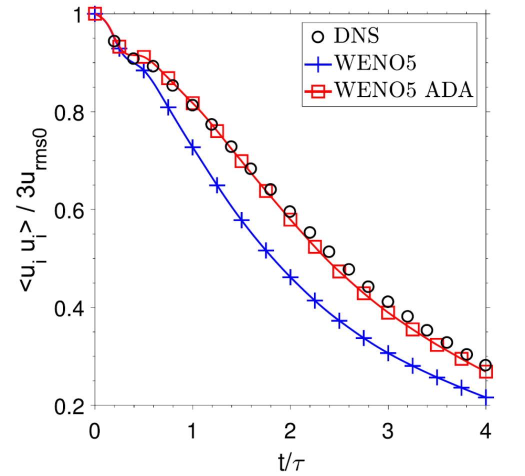
$$M_t = 0.6 \quad \chi = 0 \quad Re_\lambda = 100$$

$$\left(\rho'_{\mathrm{rms}}\right)^2 / \langle\rho\rangle^2 = 1$$
$$\left(T'_{\mathrm{rms}}\right)^2 / \langle T\rangle^2 = 1$$

- Domain: $[0, 2\pi]^3$
- Final time of simulation: $t/\tau = 4.0$
- Number of points in mesh: 64^3

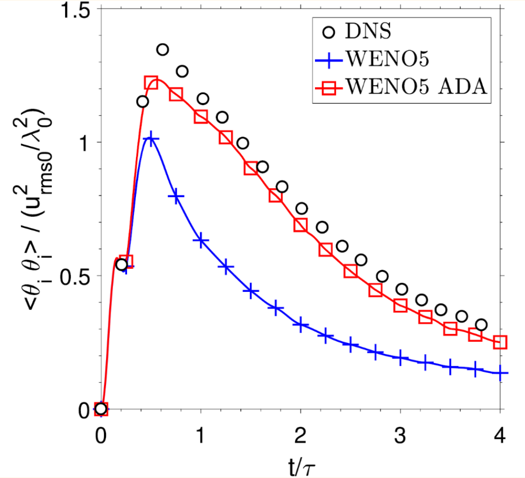
DNS: E. Johnsen *et al.*, in JCP, vol. 229(4), 1213–1237, (2010).

Decay of compressible homogeneous isotropic turbulence. HIT



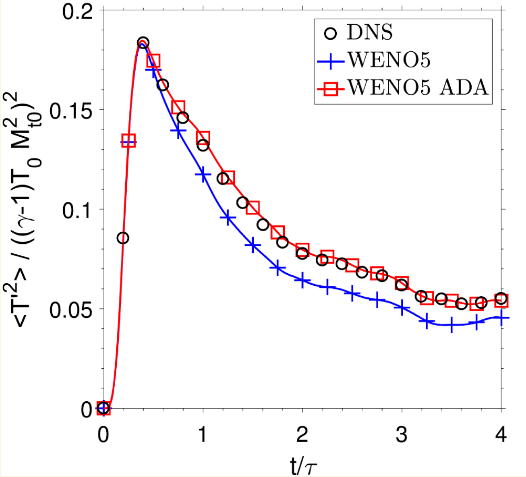
Mean-square velocity time evolution

Decay of compressible homogeneous isotropic turbulence. HIT



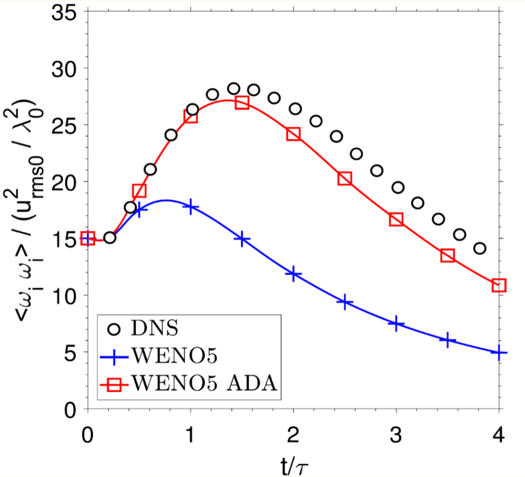
Dilatation time evolution

Decay of compressible homogeneous isotropic turbulence. HIT



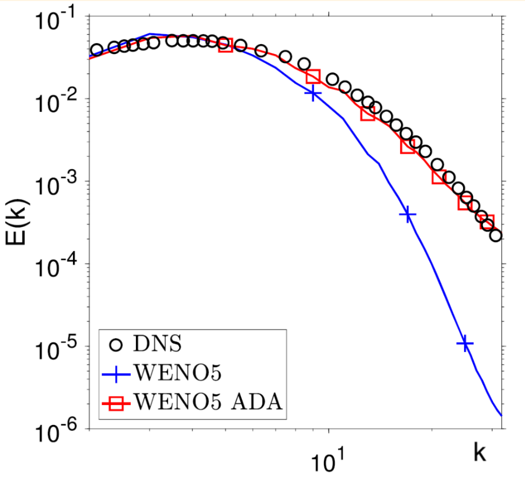
Normalized mean-square temperature fluctuations time evolution

Decay of compressible homogeneous isotropic turbulence. HIT



Enstrophy time evolution

Decay of compressible homogeneous isotropic turbulence. HIT



Instantaneous three-dimensional velocity spectrum at $t/\tau = 4$ for a 64^3 grid

SPH example:2D decay turbulence

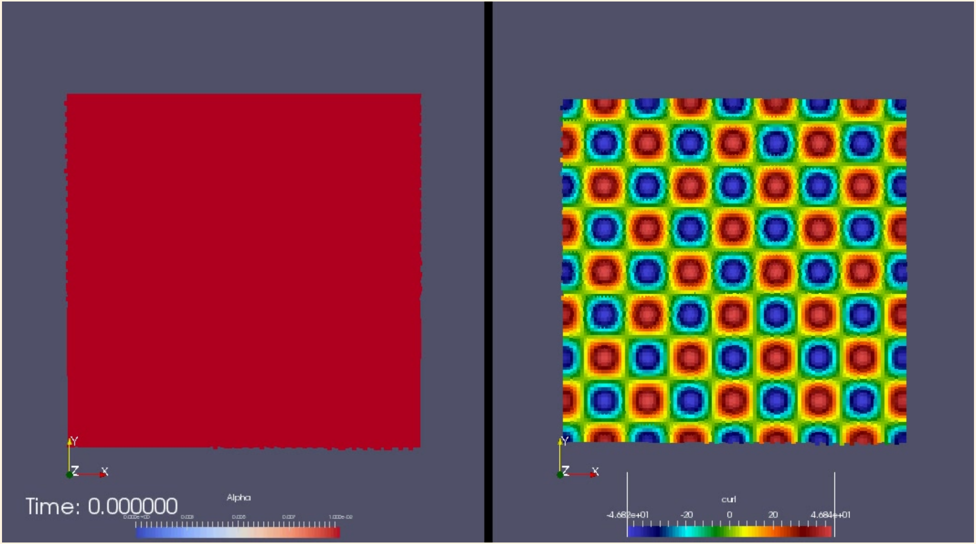
- We check the evolution of the 2D inviscid Taylor-Green Vortex
- The particles were placed initially on a grid of squares in motion with velocities specified by a 8×8 array of Taylor-Green vortices

$$
\begin{aligned}
u(x,y) &= -U_0 \cos(8\pi x)\sin(8\pi y) \\
v(x,y) &= U_0 \sin(8\pi x)\cos(8\pi y)
\end{aligned}
$$

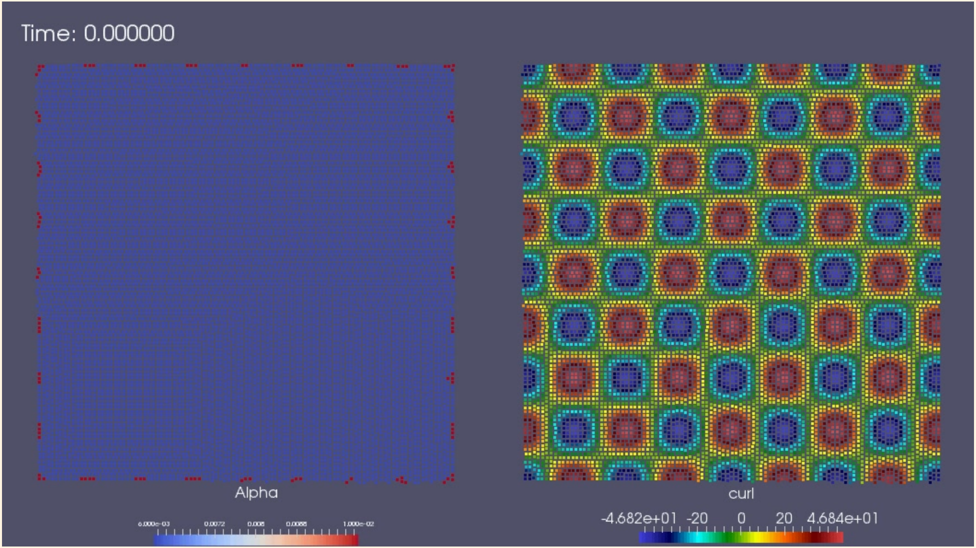
- The initial position of the particles is computed using the Particle packing algorithm [1]

[1]: A. Colagrossi *et al.*, in Computer Physics Communications, vol. 183, pp. 1641-1653, 2012

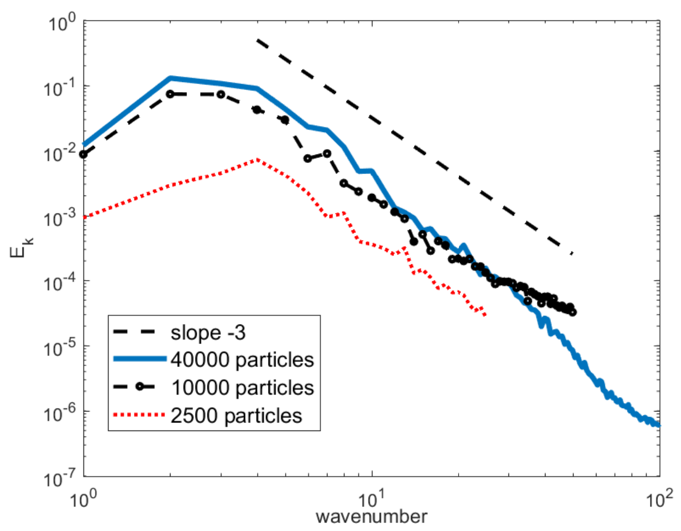
SPH example:2D decay turbulence



SPH example:2D decay turbulence

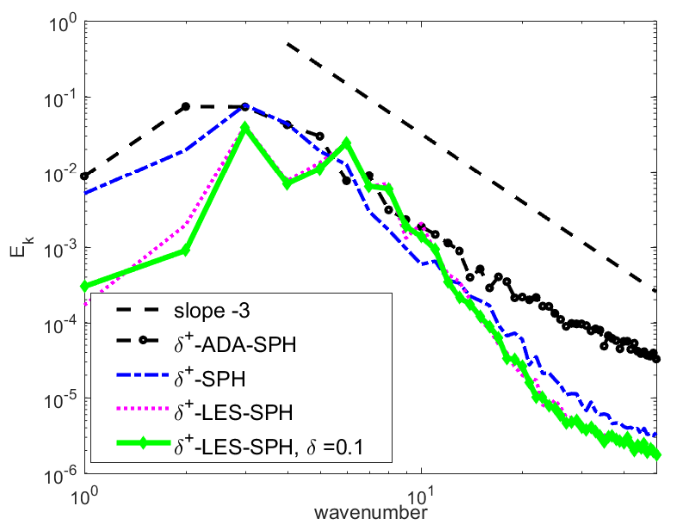


SPH example:2D decay turbulence



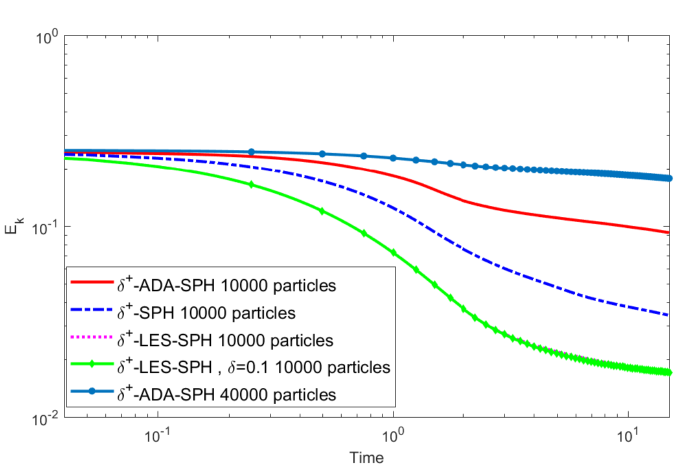
Energy spectrum for several discretizations

SPH example:2D decay turbulence



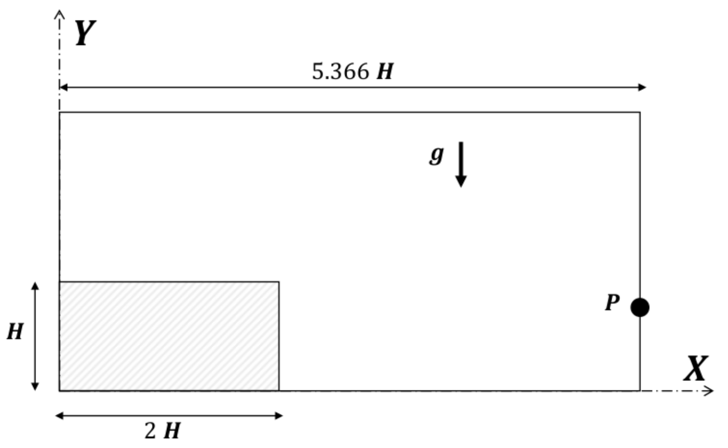
Energy spectrum using different methods using 10000 particles

SPH example:2D decay turbulence



Evolution of the kinetic energy using different schemes and different particle resolutions.

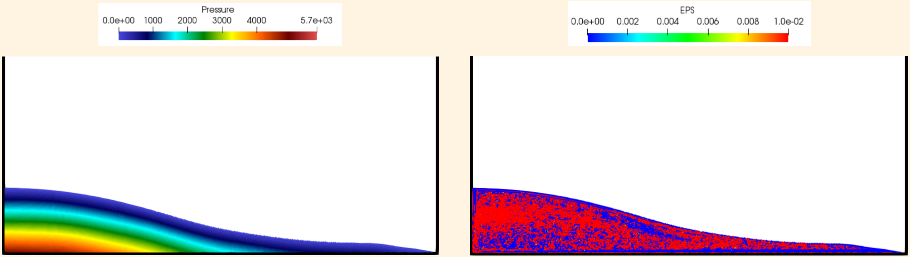
SPH example:2D dam break



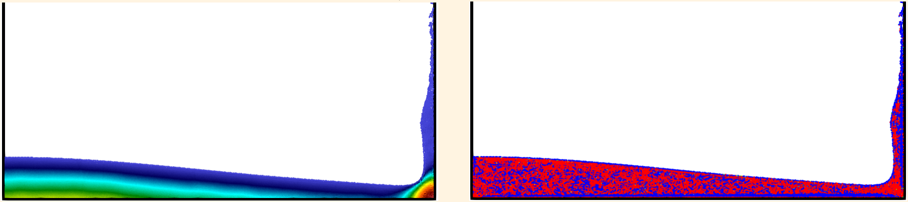
Initial setup of the 2D dam break problem.

- Pressure field is initialized from the solution of a Poisson equation on the water column

SPH example:2D dam break

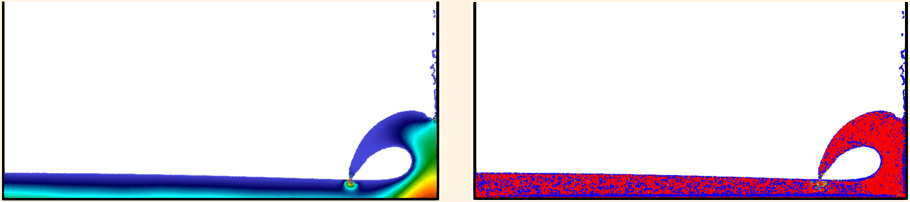


$$t\sqrt{g/H} = 2.35$$

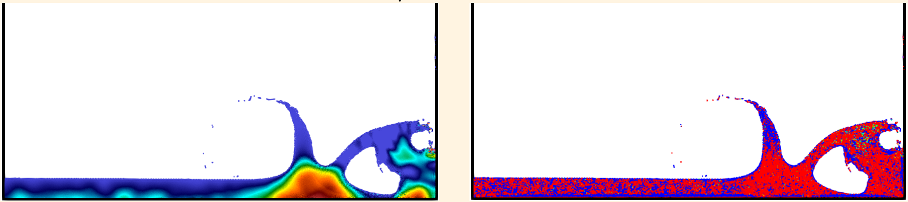


$$t\sqrt{g/H} = 4.04$$

SPH example:2D dam break

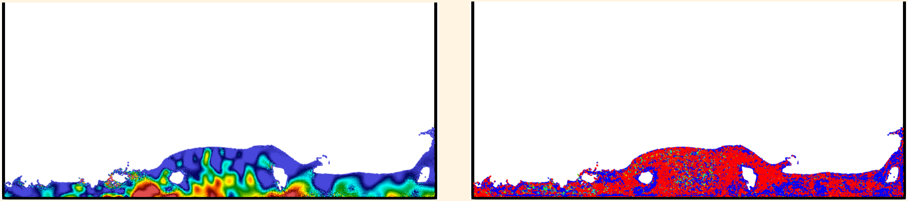


$t\sqrt{g/H} = 6.07$

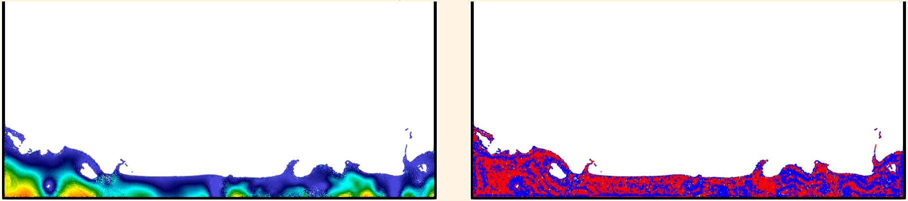


$t\sqrt{g/H} = 7.12$

SPH example:2D dam break

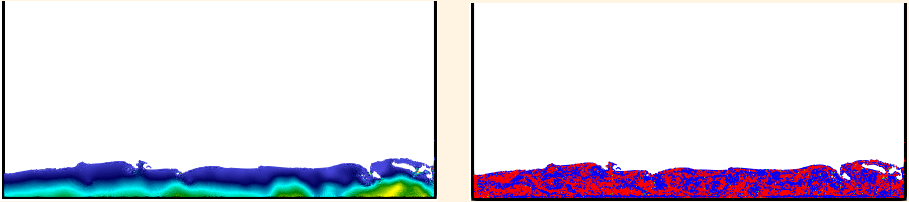


$$t\sqrt{g/H} = 9.54$$

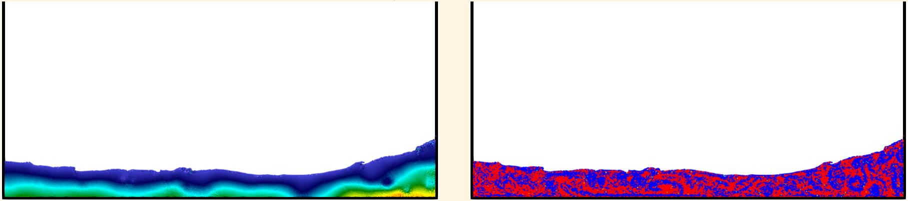


$$t\sqrt{g/H} = 13.02$$

SPH example:2D dam break

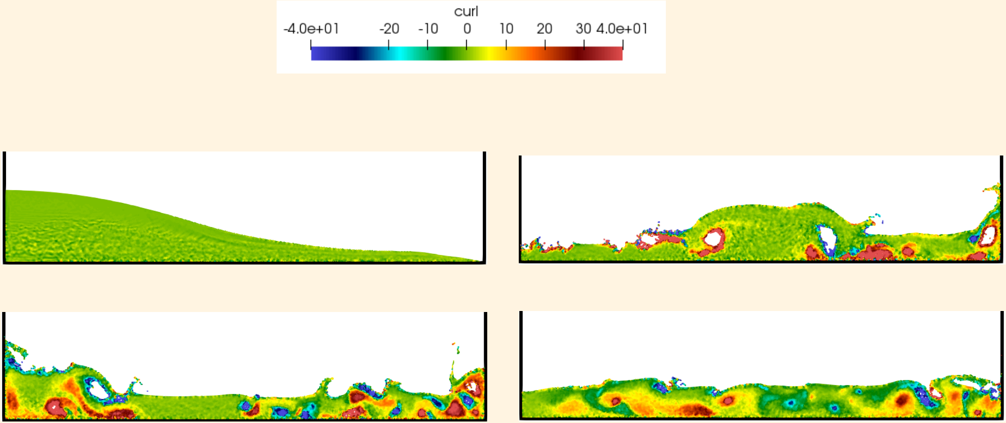


$$t\sqrt{g/H} = 19.09$$



$$t\sqrt{g/H} = 21.03$$

SPH example:2D dam break



Vorticity field obtained with $H/\Delta x_0 = 120$

Overall conclusions

Conclusions

Adaptive viscosity methods for the computation of turbulent flows

A new adaptive dissipation strategy is applied to compressible turbulence is presented.

- The scheme is modified with a multiplicative coefficient $\varepsilon \in [0, 1]$ that regulates the introduced dissipation by taking into account the high-frequency content of the solution.
- The high frequencies are measured via the *energy ratio*, obtained by filtering the velocity field with two filters of different width.
- The solutions obtained with the proposed scheme are stable due to the *a posteriori* approach and possess physical meaning, since they closely follow the DNS results.
- The proposed method is applied to the δ^{+} SPH method. Possible extension to meshless methods with Riemann solvers [1].

[1]: X. Nogueira *et al.*, High-accurate SPH method with Multidimensional Optimal Order Detection limiting, Computer Methods in Applied Mechanics and Engineering, 310, 134-155, 2016.

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  - Grant PID2021-125447OB-I00 funded by MCIN/AEI/ 10.13039/501100011033 and by "ERDF A way of making Europe".
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  - *Universidade da Coruña (UDC)*
  - *Grupo de Métodos Numéricos en Ingeniería - GMNI*

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